3-2 Videos Guide

3-2a

- Definition of a limit of a function f of two variables
  - $\lim_{(x,y)\to(a,b)} f(x,y) = L$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that if (x,y) is in the domain of f and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then $|f(x,y) - L| < \varepsilon$

Exercises:

3-2b

- Find the limit, if it exists, or show that the limit does not exist
  - $\lim_{(x,y)\to(2,-1)} \frac{x^2y+xy^2}{x^2-y^2}$
  - $\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 x}{x^4+y^4}$

3-2c

  - $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4}$

- Reminder of the Squeeze Theorem in $\mathbb{R}^2$ (which also holds in higher dimensions)
  - If $f(x) \leq g(x) \leq h(x)$ on an open interval containing a (except possibly at a) and if $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$

Exercise:

- Determine the set of points at which the function is continuous.
  $G(x,y) = \ln(1+x-y)$
- Use polar coordinates to find the limit.
  $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$